

# Assessing the risk of Terrorism using Extreme Value Statistics

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# Terrorism risk from several perspectives

## 1. The insurance industry

- Government role:
  - Terrorism Risk Insurance Act of 2002 (TIRA): A temporary measure to make risk insurance available
- Valuation of risk: A key challenge
  - A Probabilistic approach to risk valuation has met with resistance
  - Use of Extreme Value Statistics: The possibility of using Extreme Value Statistics for risk valuation has been acknowledged (e.g., Kunreuther 2002)

## 2. Finance and Reinsurance Industry

- Cat bonds (Catastrophic bonds)
  - For example see the GAO testimony before a House Subcommittee, October 8, 2002)

## 3. Economics

- Behavioral Game Theory
  - Analyzes the strategic interactions of terrorists adjusting their behavior to the defensive acts by anti-terror groups (police, military, etc.) (e.g., T. Sandler and W. Enders, 2004, J, Major, 2001, others)

# Terrorism risk and extreme value theory

- The first two approaches invoke the need view terrorism risk in probabilistic terms.
- Is terrorism a stochastic or a deterministic event?

# Three reasons why terrorism can be viewed as a stochastic event

1. Terrorism is a classic case of asymmetric information between the terrorist and his victims: intentional and deterministic to the terrorist, but probabilistic to its victims. **Deterministic at the micro level but random at the macro level.**

Example: explaining criminal behavior versus crime statistics

2. Counter terrorism may succeed  $X$  fraction of time and fail  $1-X$  fraction of time. Thus, a successful attack represents a random “system failure” (in the sense of operations management)

Example: The counter-terrorist activities in Europe over the past several years succeeded in disrupting numerous terrorist plots, but failed in the instances of Madrid and London (PBS Frontline, July 05).

### 3. The “complex systems” approach advanced by mathematical physicists (Johnson and Spagat).

Consider, “a group of self-contained ‘attack units’, each of a particular strength. Such units can join together or fragment into smaller pieces. Over time, an equilibrium of joining and breaking is reached, but where that equilibrium lies depends on the strength of any central organization.’ (Economist, July 21, 2005)

Their approach leads to the statistical view of terrorism that is similar to ours:

# Europe Vs US:

## Differing or Converging Perspectives on the Probabilistic perspective?

## 2003 (Differing Perspectives):

- “Whereas **U.S.** homeland security strategy is now driven by **potential consequences**, **European** security tends to pivot on **probabilities...**” Jonathan Stevenson (Foreign Affairs, March/April 2003)

# 2005 (Converging Perspectives) :

Excerpts from Remarks by Secretary Chertoff, (Washington, D.C. July 13, 2005):

- “...DHS must base its work on **priorities** that are driven by risk. Our goal is to maximize our security, **but not security ‘at any price.’** ”
- “We must increase preparedness with particular focus on **catastrophic events**”
- “Although we have substantial resources to provide security, **these resources are not unlimited**. Therefore, as a nation, we must make tough choices about how to invest finite human and financial capital to attain the optimal state of preparedness. To do this we will focus preparedness on **objective measures of risk** and performance.”

Since risk management involves assessing probabilities of catastrophic events, US and European approaches seem to be merging.

# Who benefits from terrorism risk assessment and how?

- Both the public and private sector
  - By enabling decision makers to evaluate the expected benefits from engaging in activities and investments that lower the risk of terrorism.
  - By helping to create efficient markets that could trade in instruments that rely on assessing the risk of terrorism (e.g., cat bonds).

# Big Challenge:

How does one measure the chance of an event that is **rare** in its occurrence and **extreme** in its magnitude? (several orders of magnitude larger than average).

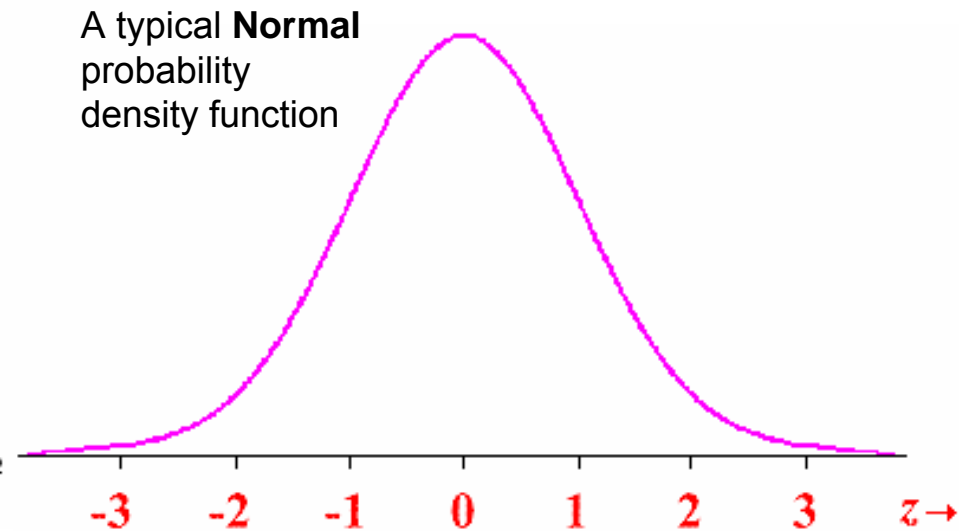
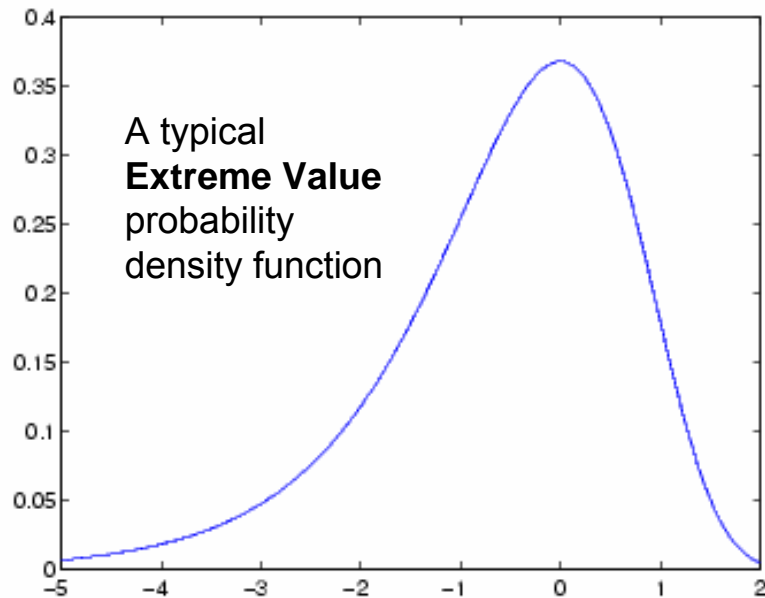
# Lets get a little in details

- If a random variable  $X$  has some distribution  $F(X)$ , knowledge of  $F(X)$  would makes calculating the probability that  $X$  exceeds some value a trivial task.
- But in general we don't know  $F(X)$ 
  - $F(X)$  could belong to any distribution class

- Can we estimate  $F(X)$ ?
- Yes and No
- Suppose our estimate of  $F(X)$  is  $G(X)$ . In general we can expect  $F(X)$  to closely approximate  $G(X)$  *but only near the mode of the distribution.*
- However *even small errors in estimating  $F(X)$  in the modal part can lead to very imprecise predictions of the tail quantiles.*
- How do we solve this dilemma?

# Fisher-Tippett theorem

- By utilizing the Fisher-Tippett theorem:
- It is a limiting theorem (similar to Central Limit Theory)
- States that the **maxima** drawn from an independent sample of draws belongs to one of only three distributions (with fatter tails than normal) **regardless** of the distribution of the original draws. These are known as the Extreme Value (EV) distributions.
- The Fisher-Tippett theorem lets us estimate the distribution of  $X_{max}$ , say  $h(X_{max})$ , from which we can predict  $Pr(X_{max} > x)$



- Note that the Extreme Value density function is skewed to the left.
- Implies that extreme events are more likely to occur than are predicted by a normal density function.

# The three classes of EV are

- Gumbel : 
$$G(x) = \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right), -\infty < x < \infty$$

- Fréchet : 
$$G(x) = \begin{cases} 0, & x \leq \mu \\ \exp\left(-\left(\frac{x-\mu}{\sigma}\right)^{-\alpha}\right), & x > \mu \end{cases}$$

- Weibull: 
$$G(x) = \begin{cases} \exp\left(-\left(\frac{x-\mu}{\sigma}\right)^\alpha\right), & x \leq \mu \\ 1, & x > \mu \end{cases}$$

- $\mu$  is called the location parameter
- $\sigma$  is called the scale parameter.

# GEV density function

- These can be generalized into a single density function, known as GEV:

$$GEV(x) = \exp\left\{-\left[1 + \xi\left(\frac{x-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}; 1 + \xi\left(\frac{x-\mu}{\sigma}\right) \geq 0$$

- GEV is known as Generalized extreme value density function.
- The parameter  $\xi$  generalizes the distribution and indicates the thickness of the “tail.” ( $\xi > 0$  means very heavy tails)
- The estimated  $\xi$  tells us which of the three distributions fits the data better.

# Existing Data

- ITARATE (International Terrorism: Attributes of Terrorist Events).
  - Best known dataset, but covers only transnational events, ignores domestic events.
- PGIS (Pinkerton Corp. Global Intelligence Services).
  - Includes all acts of terrorism, but is not publicly available. (coded through U. of Maryland)
- Monterey Institute's Center for Nonproliferation Studies (CNS).
  - Has compiled 1200 incidents going back to 1900 of CBRN (chemical Biological, Radiological and Nuclear) incidents, but includes many hoaxes and pranks. Not publicly available.
- MIPT (National Memorial Institute for the Prevention of Terrorism) Terrorism Knowledge Base.
  - Over 21,000 observations of both domestic and international terrorism. The data *are* public domain, but the dataset is not public. Instead each incident is posted on a *separate* webpage. The set includes only 56 biological and chemical incidents.

# Our data

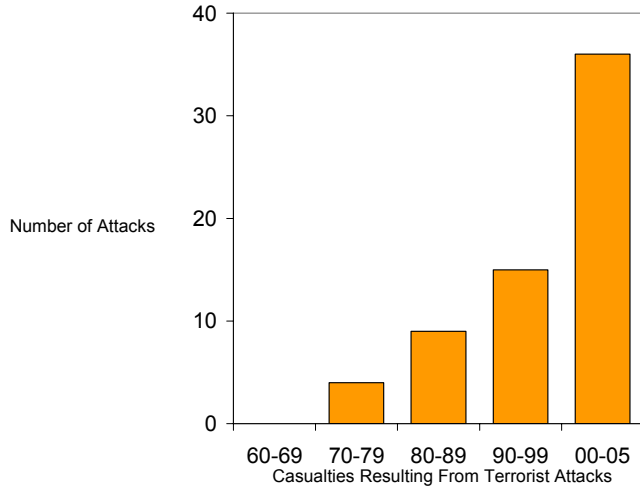
- 314 incidents of direct use of chemical nuclear, biological radioactive or nuclear agents (CBRN), or implied threat to their containment, by a group or individual.
- Compiled from many different sources. (see paper for details)

# The Most Serious Attacks on Food Chain: 1961-2005

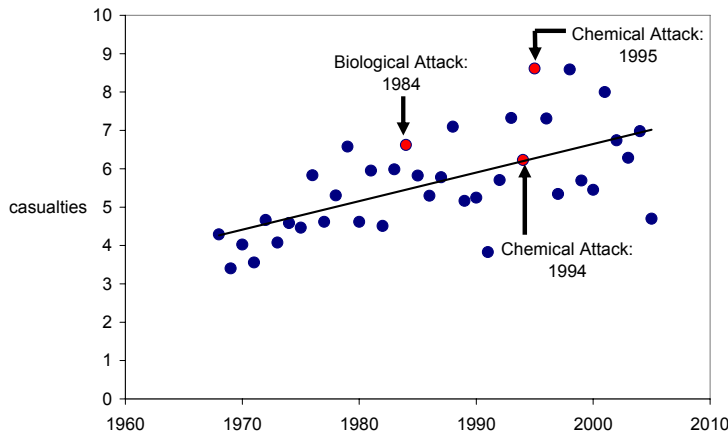
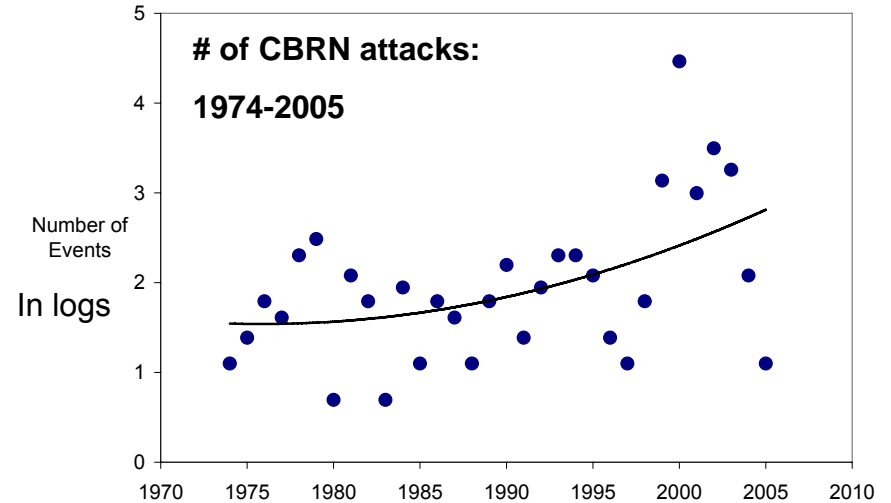
11/19/2003	0	50	About fifty people in more than 20 cities in Italy had to be treated for a variety of ailments including stomach pains, after they drank bottled water that had been injected with either bleach acetone or ammonia.
10/1/2003	0	64	Cao Qianjin threw 500 ml of a pesticide into the reservoir in Ruyang County, Henan. Province, China. Approximately 64 residents were poisoned.
9/23/2003	0	241	Several hundred (317) students and staff at an elementary school in Yueyang, Hunan Province, China, were sent to hospitals, after eating breakfast that had been laced with rat poison. Investigators stated that 241 students and staff showed some signs of poisoning.
12/31/2002	0	111	A supermarket employee in the US poisoned about 250 pounds of ground beef with an insecticide. At least 111 people who fell ill after eating the meat.
June, 2002	0	60	In June 60 students and teachers at a school in Volgograd, Russia, were hospitalized after being poisoned with the salmonella typhi toxin.
5/19/2002	7	47	Seven members of the Johanne Marange Apostolic Church, a Christian fundamentalist group, died and another 47 were taken ill after drinking a tea that had been poisoned.
1/30/2002	unknown	92	In Linxiang city, in China's Hunan Province, 92 children at the Yucai Private (primary) School fell ill after eating their school lunch which had been laced with rat poison. Of the 92, 40 were in serious condition.
8/8/2001	0	120	At least 120 patrons in 16 restaurants were made ill after eating noodles that had been contaminated with rat poison in Ningxiang, Hunan Province, China.
3/8/2000	2	60	Poisoned food was served to hundreds of students at a religious school in Jalaludin, Afghanistan. Two students died and sixty others lost consciousness.
11/3/1999	unknown	48	Approximately 48 people fell ill after eating meat rolls that had been laced with rat poison at a fast food restaurant in Deyang City, Sichuan Province, China.
3/8/1999	0	148	Five people were arrested in China, after putting nitric acid in a popular restaurant's specialty donkey meat soup, poisoning 148 people.
7/25/1998	4	60	Four people died and approximately 60 were hospitalized during a summer festival in Wakayama, Japan, in a case of mass food poisoning that possibly involved the use of cyanide.
1/1/1994	15	53	On New Year's day nine Russian soldiers and at least six civilians died after drinking champagne that had been laced with cyanide. The cyanide-laced champagne was being sold outside of the Russian compounds. Another 53 people, including 11 civilians, were hospitalized.
9/6/1987	19	140	Several fatalities resulted when members of the Philippine Constabulary were poisoned after accepting bags of ice water from an individual during a "fun run." As a result 19 people died and 140 fell ill.
Sep. 1984	0	751	Sometime early in September 1984, members of a religious cult known as the Rajneeshees contaminated salad dressing at ten restaurants in a small town in Oregon, USA. As a result 751 people became sick, there were no fatalities.

# Trends in the incidence of Terrorism

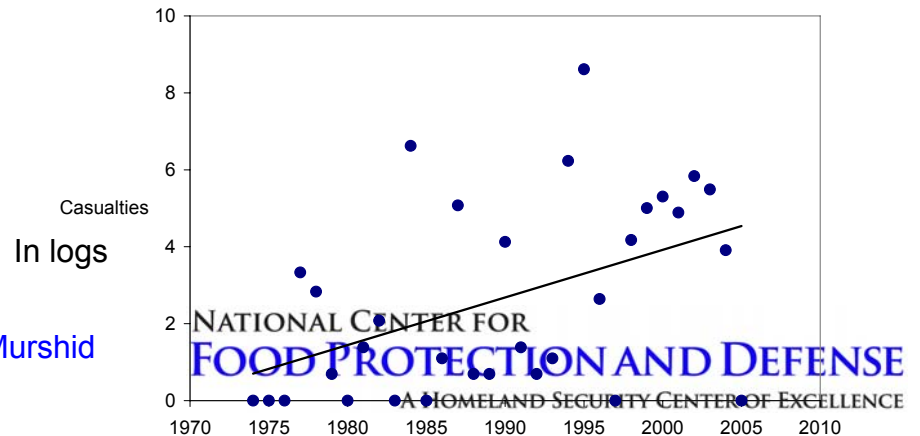
**Frequency of Attacks on Food or Water Supply**



Number of CBRN Events 1974-2005



**Max # of casualties on a single day from CBRN attacks: 1974-2005**  
Maximum Number of Casualties on a Single Day 1974-2005



**Source: authors' data and MIPT data**

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NATIONAL CENTER FOR FOOD PROTECTION AND DEFENSE  
A HOMELAND SECURITY CENTER OF EXCELLENCE

# Model Estimation using EVT

Since there is a **trend** in the data, estimating the probabilities must take account of this.

Thus, we must make allowance for parameters of the distribution to depend on time.

Let  $X_t$  = the maximum # of casualties in a CBRN incident. Then,

$$X_t \sim GEV(\mu(t), \sigma(t), \xi)$$

# Model to be Estimated

# GEV Parameter Estimates, Fitted to CBRN-Data

Model 1:  $\mu(t) = \beta_0, \sigma(t) = \gamma_0;$

Model 2:  $\mu(t) = \beta_0 + \beta_1 t, \sigma(t) = \gamma_0;$

Model 3:  $\mu(t) = \beta_0, \sigma(t) = \gamma_0 + \gamma_1 t;$

Model 4:  $\mu(t) = \beta_0 + \beta_1 t, \sigma(t) = \gamma_0 + \gamma_1 t;$

		Stationary	Nonstationary in location	Nonstationary in scale	Nonstationary in location and scale
location	$\beta_0$	0.78 (0.57)	0.21 (0.78)	0.85 (0.43)	-0.09 (0.02)
	$\beta_1$		0.09 (0.05)		0.09 (0.02)
scale	$\gamma_0$	1.20 (0.54)	1.90 (0.32)	1.05 (0.48)	-0.10 (0.02)
	$\gamma_1$			0.02 (0.04)	0.10 (0.02)
shape	$\xi$	0.87 (0.87)	-0.10 (0.19)	0.53 (0.75)	0.43 (0.19)
likelihood		-70.31	-69.36	-70.10	-27.43
LR Test		NA	(0.17)	(0.52)	(0.00)

Notes: Estimation was done in R using the ISMEV package.

The ISMEV package is based on software written by Stewart Coles.

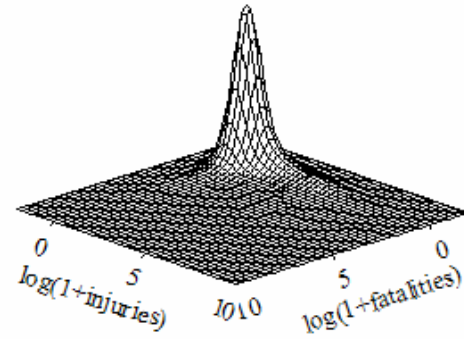
Standard errors are reported in parentheses.

The LR test reports the p-value for the null hypothesis that the model parameters are constant.

- Similar results are found by estimating the parameters of a GEV for MIPT data
- These results were further verified by several diagnostic tests (Q-Q plots, residual probability PP plots, etc.)

# The challenge of outliers

- If we combine death and injury there is some structure in the tails of the distribution reflecting for example the sarin attacks in Tokyo and few other cases.

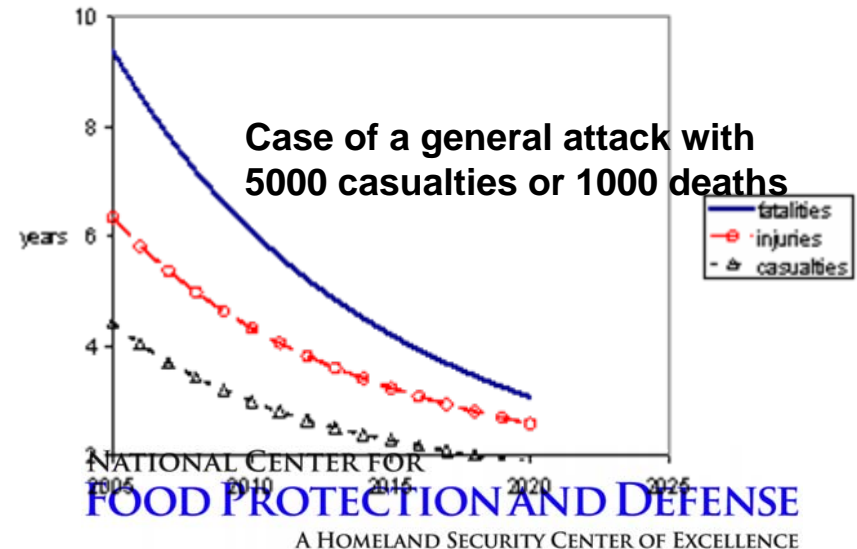
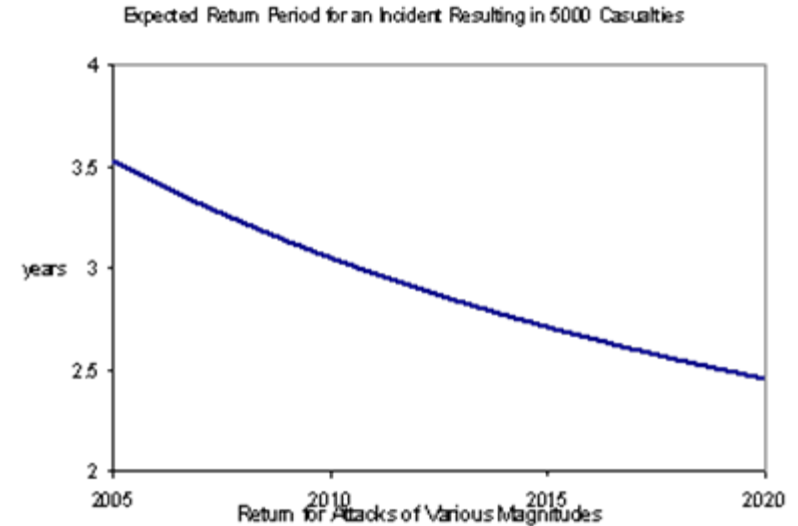


Note: there is a fatter tail along the injury axis than fatality axis

# Recurrence Period of terrorist events

- Based on our estimated model, we calculate the expected “reoccurrence period” for a catastrophic event.
- **For a CBRN Attack:**
- An event of the scale of Tokyo subway attack (with 5000 casualties) could occur by 2009.
- Note that reoccurrence period is decreasing with time. By 2020 an attack of that magnitude could occur about every 2 ½ years.
- **For a General Attack:**
- The results are even more extreme

## Case of CBRN attack with 5000 casualties



# Effect of Policy Intervention

- Note that these estimates assume a continuation of current trends, which may or may not be true.
- What about the effect of intervention?
- We have done a similar EV analysis on the spread of BSE in UK and find the effect of intervention to be very significant.
- We should be able to include the effect of our more proactive approach in the US in a few years. Too early to tell yet.

# Extensions and Remaining Issues

- We are **adding new data** (we now have near 600 data on CBRN observations)
- More data will allow for breakdown into more agent-specific risk analysis. Yet, don't want to become too agent-specific!!  
**Substitutability** in terrorist targets.
- Some technical Issues to be addressed: **conditional** probabilities:
  - What is the chance of a specific type of attack (e.g., food), given that an attack would occur?
  - This question has application to other areas of DHS concern, e.g., transportation and logistics, infrastructure, etc.
  - But doing conditional probabilities with extreme value distribution is tricky.

# Extensions and Remaining Issues - Continued

- Application to the risk of spread of diseases
  - We have applied this approach to BSE data in UK. While BSE may be a good example of how public authorities may respond to crisis, it is **not** the best case to study **as analogy** to a terrorist or catastrophic event because of its **slow** spread and the **stability** of the agent.

# Extensions and Remaining Issues - Continued

- What are the best analogies to study?
  - The event must have the potential to lead to crisis, e.g. large economic or human impact of a rapidly spreading disease (so they simulate terrorist or catastrophic scenarios)
  - Must have data on the event, so we can study it.
  - Must have data on health policy response so we can see how effective policy has been in stopping it.
  - The agent must mutate in short periods of time, so it can simulate the adoptability of terrorists.
- A few potential candidates are:
- SARS? Bird Flu? FMD?

# Conclusion

- Acts of terrorism **can** be viewed as probabilistic events.
- Such probabilities **can** be measured even if they occur seldom and are of extreme type.
- Measuring these probabilities is a first step towards rationalizing the use of scarce resources in defending against rare but catastrophic risks such as terrorism.