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Myopic Agents and Interdependent Security Risks: A Comment on “Interdependent Security” by Kunreuther and Heal

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Abstract

We present equilibrium strategies for agents in an interdependent security model in which threats occur over time. We explore the effects of differing discount rates on the nature of the equilibrium solutions, and the security strategies adopted by the agents.

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Kunreuther and Heal (2003, 2005) discuss an interdependent security model in which agents are subjected both to direct attacks (which can be prevented by investment in security), and to indirect attacks, due to “contagion” from direct attacks on other agents (which can be prevented only by the actions of those other agents). However, the model of Kunreuther and Heal is static, in the sense that all attacks are assumed to occur at a single instant in time. Such a model does not adequately address real-world situations in which attacks happen over time. In a time-dependent model, protection against a subset of attacks (such as direct attacks) would postpone the expected time until an attack, but might not decrease the probability of succumbing to an attack eventually. This suggests that agents’ discount rates may play a key role in determining equilibrium investment strategies. Thus, a revised analysis is needed to explain the effects of time preferences on the part of the agents in the model. In particular, this allows us to investigate the effects of heterogeneity in discount rates.

We find, unsurprisingly, that agents with high discount rates would not invest in security. More significantly, we find that even agents with lower discount rates may not find it worthwhile to invest in security unless other agents also do so. Thus, a high discount rate on the part of one agent (e.g., due to myopia) can make it undesirable for other agents to invest when that would otherwise be optimal.

1. Our Revised (Time-Dependent) Model

Consider two agents, each having its own computer system. The two systems are connected to each other, and also to the outside world through the Internet. Viruses can attack each system, either directly through the Internet, or indirectly through an attack on the other system. We assume that the times of direct and indirect attacks are independent, and that even a single attack on a computer system would be catastrophic to the system owner, so that the effects of subsequent attacks need not be considered. We also assume that the time (t_i) of a direct attack on system *i* follows an exponential distribution with parameter λ_i for *i* = 1, 2. The probability that a direct attack on system *i* leads to an indirect attack on the other system is denoted by q_i. The cost of investing in security for agent *i* is denoted by C_i; this investment is assumed to occur at time 0, and is assumed to eliminate the risk of direct attacks on system *i,* but to have no effect on the risk of indirect attacks. The discount rate of agent *i* is denoted by r_i, and the loss suffered by agent *i* if it is attacked (either directly or indirectly) is denoted by L_i.

One can show that the expected net present value of the loss experienced by agent 1 is $\frac{L_1(\lambda_1+\lambda_2 q_2)}{\lambda_1+\lambda_2 q_2+r_1}$, and similarly $\frac{L_2(\lambda_2+\lambda_1 q_1)}{\lambda_2+\lambda_1 q_1+r_2}$ for agent 2. Our results regarding the effects of the parameters λ_i, q_i, C_i, and L_i (where *i* = 1, 2) on the Nash equilibrium solutions to this game are qualitatively comparable to those obtained by Kunreuther and Heal, and hence are not reported here. Instead, we focus here on the novel feature of our model – namely, the effect of the discount rates r_i. For simplicity, we assume that $\lambda_1=\lambda_2=\lambda$, $q_1=q_2=q$, $C_1=C_2=C$, and $L_1=L_2=L$. The payoff matrix for this case is shown in Table 1. There are four possible cases, as discussed below:

Table 1 goes here

Case 1. Both agents invest in security (S, S) will be the unique equilibrium strategy when the cost of investing in security is low for both agents; i.e.,

$$C<\underset{i=1,2}{\mathrm{Min}}\left[\frac{L\lambda}{\lambda+r_i}\right], \text{ and also } C<\underset{i=1,2}{\mathrm{Max}}\left[\frac{\mathrm{L}\lambda r_i}{(\lambda+\lambda q+r_i)(\lambda q+r_i)}\right]$$

Case 2. Neither agent invests in security (N, N) will be the unique equilibrium strategy when the cost of investing in security is high for both agents; i.e.,

$$C > \operatorname*{Max}_{i=1,2}\left[\frac{L\lambda r_i}{(\lambda+\lambda q+r_i)(\lambda q+r_i)}\right], \text{ and } C > \operatorname*{Min}_{i=1,2}\left[\frac{L\lambda}{\lambda+r_i}\right]$$

Case 3. One agent invests but the other does not invest, (N, S) or (S, N), will be a unique Nash equilibrium strategy when the cost of investing in security is high for one agent, but low for the other agent; i.e.,

$$C > \frac{L\lambda}{\lambda+r_i} \text{ and } C < \frac{L\lambda r_j}{(\lambda+\lambda q+r_j)(\lambda q+r_j)}, \text{ where } i \neq j$$

In this situation, the agent with the higher discount rate will prefer not to invest in security, while the agent with the lower discount rate will prefer to invest.

Case 4. Both agents either invest or don't invest in security, (S, S) or (N, N). This occurs when the cost of investing in security is intermediate for both agents; i.e.,

$$\frac{L\lambda r_i}{(\lambda+\lambda q+r_i)(\lambda q+r_i)} < C < \frac{L\lambda}{\lambda+r_i} \qquad \text{for } i = 1, 2$$

In this case, neither agent has a dominant strategy, and each agent prefers to adopt whichever strategy is chosen by the other agent. Investment in security will be the socially optimal decision when the combined payoffs of agents 1 and 2 for solution (S, S) are greater than the combined payoffs of agents 1 and 2 for solution (N, N); i.e., when

$$C < \frac{L(\lambda+\lambda q)}{2}\left(\frac{1}{\lambda+\lambda q+r_1}+\frac{1}{\lambda+\lambda q+r_2}\right)$$

2. Implications of the Results Regarding Discount Rates

For simplicity, we define

$$\rho_a = \frac{\lambda}{2}\left\{\left(\frac{L}{C}-1\right)-2q-\sqrt{\left(\frac{L}{C}-1\right)^2-4\frac{qL}{C}}\right\},$$

$$\rho_b = \frac{\lambda}{2}\left\{\left(\frac{L}{C}-1\right)-2q+\sqrt{\left(\frac{L}{C}-1\right)^2-4\frac{qL}{C}}\right\}, \text{ and}$$

$$\rho_c = \lambda\left(\frac{L}{C}-1\right) \text{ where } \rho_a < \rho_b < \rho_c$$

Table 2 summarizes all possible equilibrium strategies as a function of the agents' discount rates r_i. When agent i has a high discount rate $(r_i > \rho_c)$, future losses due to attacks will have low present value, so agent i will not find it worthwhile to invest in security. When agent i has a moderately small discount rate $(\rho_a < r_i < \rho_b)$, the losses due to future attacks will tend to loom relatively large, so agent i will find investing in security to be worthwhile.

When the discount rate of agent i is in the intermediate range $(\rho_b < r_i < \rho_c)$, agent i will be ambivalent about whether to invest. If the other agent decides to invest (and hence eliminates the risk of indirect losses to agent i), investing will become cost effective for agent i, since investing will now eliminate all risk, rather than only a subset of the total risk. However, if the other agent decides not to invest, investing will no longer be cost effective for agent i.

Finally, when the discount rate of agent i is extremely small $(r_i < \rho_a)$, then eliminating only a subset of the total risk is no longer worthwhile, since it only postpones the loss from an attack, rather than eliminating it; the extremely low discount rate means that merely postponing the loss is of little value to the agent. Therefore, it is again worthwhile for agent i to invest in security only if the other agent also invests.

(Table 2 goes here)

3. Conclusions

The above analysis shows that a high discount rate on the part of one agent can make it undesirable for other agents to invest in security. This is because investments in security by the agents can no longer eliminate all risk of attack (since an indirect attack will still be possible), and instead merely postpones the timing of an eventual attack.

Moreover, under some circumstances, there are multiple equilibrium solutions to the game considered here (all agents invest, or no agents invest), creating a risk that the social optimum may not be reached. Coordination mechanisms such as contractual agreements can help to ensure that the social optimum is reached (Kunreuther and Heal 2003, 2005).

Agents may have different discount rates for a variety of reasons. First, firms in different industries (with differing levels of risk) or with differing conditions of their physical plant (and hence different opportunities for investing in modernization) will tend to have differing minimum acceptable rates of return. Secondly, firms facing impending bankruptcy may have high discount rates, and be willing to take much greater risks than firms that are financially stable. Finally, of course, agents may simply be myopic (Kunreuther et al., 1998), in the sense of adopting higher effective discount rates and shorter time horizons than would be in their own (enlightened) self-interest. It is noteworthy that such myopia can eliminate the incentives for non-myopic agents to invest in security.

Extending the model of Kunreuther and Heal (2003, 2005) to one in which attacks occur stochastically over time showed that heterogeneity in discount rates can complicate the task of achieving security in the face of externalities. Recognition of this phenomenon will hopefully make it possible to craft appropriate coordinating mechanisms to address this challenge.

References

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Table 1. Payoff matrix

	Agent 2 invests	Agent 2 does not invest
Agent 1 invests	$-C;\ -C$	$-C-\frac{L\lambda q}{\lambda q+r_1};\ -\frac{L\lambda}{\lambda+r_2}$
Agent 1 does not invest	$-\frac{L\lambda}{\lambda+r_1};\ -C-\frac{L\lambda q}{\lambda q+r_2}$	$-\frac{L(\lambda+\lambda q)}{\lambda+\lambda q+r_1};\ -\frac{L(\lambda+\lambda q)}{\lambda+\lambda q+r_2}$

Table 2. Equilibrium strategies as a function of discount rates

r_1 \ r_2	**Extremely Small** $0 < r_2 \leq \rho_a$	**Moderately Small** $\rho_a < r_2 < \rho_b$	**Intermediate** $\rho_b \leq r_2 \leq \rho_c$	**High** $r_2 \geq \rho_c$
Extremely Small $0 < r_1 \leq \rho_a$	(S, S) or (N, N) Multiple Nash equlibria	(S, S) Nash equilibrium	(S, S) or (N, N) Multiple Nash equlibria	(N, N) Nash equilibrium
Moderately Small $\rho_a < r_1 < \rho_b$	(S, S) Nash equilibrium	(S, S) Dominant strategy	(S, S) Nash equilibrium	(S, N) Nash equilibrium
Intermediate $\rho_b \leq r_1 \leq \rho_c$	(S, S) or (N, N) Multiple Nash equlibria	(S, S) Nash equilibrium	(S, S) or (N, N) Multiple Nash equlibria	(N, N) Nash equilibrium
High $r_1 \geq \rho_c$	(N, N) Nash equilibrium	(N, S) Nash equilibrium	(N, N) Nash equilibrium	(N, N) Dominant strategy