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A MODELING FRAMEWORK FOR FACILITY LOCATION OF MEDICAL SERVICES FOR LARGE-SCALE EMERGENCIES

Jia, H., Ordóñez, F. & Dessouky, M.

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**Center for Risk and Economic Analysis of Terrorism Events
University of Southern California
Los Angeles, California**



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Hongzhong Jia, Fernando Ordóñez, and Maged
Dessouky
Epstein Department of Industrial and Systems
Engineering
University of Southern California

Abstract

Research on facility location is abundant. However, this research does not typically address the particular conditions that arise when locating facilities to service large-scale emergencies, such as earthquakes, terrorist attacks, etc. In this work we first survey general facility location problems and identify models used to address common emergency situations, such as house fires and regular health care needs. We then analyze the characteristics of large-scale emergencies and propose a general facility location model that is suited for large-scale emergencies. This general facility location model can be cast as a covering model, a P -median model or a P -center model, each suited for different needs in a large-scale emergency. Illustrative examples are given to show how the proposed model can be used to optimize the locations of facilities for medical supplies to address large-scale emergencies in the Los Angeles area.

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1. Introduction

Research on facility location problems is abundant and many models have been developed to formulate and solve various location problems. However the prior work on facility location problems has not considered the special characteristics of large-scale emergency situations. In this paper we define large-scale emergencies as those rare events that overwhelm local emergency responders and require regional and/or national assistance, such as natural disasters (examples are the Northridge earthquake and hurricane Floyd) and terrorist attacks such as September 11. The staffing levels and materials of local emergency responders are designed to cope with the regular small-scale emergencies, such as household fires or vehicle accidents, and it is for these frequent emergencies that many facility location problems have been investigated. These solutions however do not translate well for large-scale emergencies. The tremendous magnitude and low frequency of the large-scale emergencies require a modification in the definition of facility coverage to allow for redundant facility placements and tiered facility services to ensure an acceptable form of coverage of all demand areas in case of a large-scale emergency.

Our research aims to determine the optimal emergency medical service (EMS) facility locations to address the needs generated by large-scale emergencies. Although facility location models are used to place any type of service in a network, from hospitals to fire stations to triage areas, in this work we discuss only the problem of locating medical supplies for large-scale emergencies. In particular, we concentrate on the problem of where to deploy local medical stocks such as the protective equipment and antidotes against a dirty bomb attack, and the problem of how to position local staging centers to receive, repackage and distribute the medical supplies from the strategic national stockpiles (SNS) for a major natural disaster or a bioterrorist attack.

In general, facility location problems of large-scale emergency medical services (LEMS) have to decide the number and location of facilities. Moreover, due to the low frequency and large potential impact of these events, these models should also consider:

- the appropriate strategies for the facility deployment (facility location objective),
- the number of facilities assigned to each demand point (facility quantity), and
- the distance up to which a facility should service a demand point (service quality).

The reason for ambiguity in the facility location objective is that a large-scale emergency is bound to impact lives regardless of the solution. Therefore care should be taken in prioritizing one solution over another. Another important aspect of a large-scale emergency solution is that resources in nearby areas can be pooled to address a need. Thus, further coordination of the local stocks should be allowed.

The objectives of this paper are to: (1) review the literature related to the EMS facility location problems for regular emergency situations, and (2) formulate a LEMS facility location model that is suited for large-scale emergency situations. The rest of the paper is organized as follows: Section 2 classifies traditional facility location models. In Section

3, we present a literature review of location models for EMS and other emergency services. This section is separated according to three different problem objectives that are used in traditional facility location models: covering, minsum (P -median), and minmax (P -center). In Section 4, we analyze the characteristics of large-scale emergencies and propose a general LEMS facility location model. In Section 5, we present examples that illustrate the use of the proposed model for different large-scale emergency scenarios. We finish with conclusions and directions for future work in Section 6.

2. Model Classification

2.1 Classification of Traditional Facility Location Models

Facility location models can be classified according to their objectives, constraints, solutions, and other attributes. Different classifications of facility location models for distribution systems have been proposed in the literature, for example by Klose and Drexl (2004). In what follows, we extend their discussion and present eight of the most common criteria that are used to classify the traditional facility location models.

- (1) *Topological characteristics.* Topological characteristics of the facility and demand sites lead to different location models including continuous location models (Plastria, 1987), discrete network models (Daskin, 1995), hub connection models (Campbell, 1994), etc. In each of these models, facilities can only be placed at the sites where it is allowed by topographic conditions.
- (2) *Objectives.* The objective is an important criteria to classify the location models. Covering models aim to minimize the facility quantity while providing coverage to all demand nodes or maximize the coverage provided the facility quantity is pre-specified (Chung, 1986). P -center models have an objective to minimize the maximum distance (or travel time) between the demand nodes and the facilities (Chen and Handler, 1993). They are often used to optimize the locations of facilities in the public sector such as hospitals, post-offices and fire stations. P -median models attempt to minimize the sum of distance (or average distance) between the demand nodes and their nearest facilities (Rosing et al., 1979). Companies in the private sector often use P -median models to make facility distribution plans so as to improve their competitive edge.
- (3) *Solution methods.* Different solution methods result in different location models such as optimization models and descriptive models. Optimization models use mathematical approaches such as linear programming or integer programming to seek alternative solutions which trade off the most important objectives against one another. Descriptive models, in contrast, use simulation or other approaches to achieve successively enhanced location pattern until a solution with desired degree is achieved. Combined solution methods have also been developed by extending the descriptive models with optimization techniques to address

dynamic and interactive location problems (e.g. mobile servers) which preclude the applications of traditional location models (Larson, 1974; Larson, 1975; Brandaeu and Larson, 1986).

- (4) *Features of facilities.* Features of facilities also divide location models into different kinds. For instance, facility restrictions can lead to models with or without service capacity (Pirkul and Schilling, 1991); and facility dependencies can result in models that take into account the facility cooperation or neglect it (Batta and Berman, 1989).
- (5) *Demand patterns.* Location models can also be classified based on the demand patterns (Plastria, 1997). If a model has elastic demand, then the demand in an area will vary (either increase or decrease) with different facility location decisions; while a model with inelastic demand will not vary the demand pattern due to the facility location decisions.
- (6) *Supply chain type.* Location models can be further divided by the type of supply chain considered (i.e. single-stage model vs. multi-stage model). Single-stage models focus on service distribution systems with only one stage, whereas multi-stage models consider the flow of service through several hierarchical levels (Gao and Robinson, 1994).
- (7) *Time Horizon.* Time horizon categorizes location models into static models and dynamic models. Static models optimize the system performance deciding all variables simultaneously. In contrast, dynamic models consider different time periods with data variation across these periods, and give solutions for each time period adapting to the different conditions (Erlenkotter, 1981).
- (8) *Input parameters.* Another popular way to classify the location models is based on the features of the input parameters to the problems. In deterministic models, the parameters are forecast with specific values and thus the problems are simplified for easy and quick solutions (Marianov and ReVelle, 1992). However, for most real-world problems, the input parameters are unknown and stochastic/probabilistic in nature. Stochastic/probabilistic location models capture the complexity inherent in real-world problems through probability distributions of random variables or considering a set of possible future scenarios for the uncertain parameters (Owen and Daskin, 1998).

Location models can also be distinguished based on other attributes such as single- vs. multi-product models, or pull vs. push models. Readers interested in further details are referred to the recent review papers by Hamacher and Nickel (1998) and Klose and Drexl (2004).

3 Review of Facility Location Models for Emergency Services

Although large-scale emergencies have many unique characteristics, they also share similarities with regular emergencies. In this section, we review the literature on facility location problems for regular emergency situations. This review is separated into three sections depending on the objective function of the location models, and this leads to: covering models, P -median models, and P -center models.

3.1 Covering Models for Emergency Services

Covering models are the most widespread location models for formulating the emergency facility location problems. The objective of covering models is to provide “coverage” to demand points. A demand point is considered as covered only if a facility is available to service the demand point within a distance limit. The literature on covering problems is divided into two major parts: the location set covering problem (LSCP) and the maximal covering location problem (MCLP).

LSCP is an earlier statement of the emergency facility location problem by Toregas et al. (1971) and it aims to locate the least number of facilities that are required to cover all demand points. Since all the demand points need to be covered in LSCP, regardless of their population, remoteness, and demand quantity, the resources required for facilities could be excessive. Recognizing this problem, Church and ReVelle (1974) and White and Case (1974) developed the MCLP model that does not require full coverage to all demand points. Instead, the model seeks the maximal coverage with a given number of facilities. The MCLP, and different variants of it, have been extensively used to solve various emergency service location problems. A notable example is the work of Eaton et al. (1985) that used MCLP to plan the emergency medical service in Austin, Texas. The solution gives a reduced average emergency response time even with increased calls for service. Schilling et al. (1979) generalized the MCLP model to locate emergency fire-fighting servers and depots in the city of Baltimore. In their model, known as FLEET (Facility Location and Equipment Emplacement Technique), two different types of servers need to be located simultaneously. A demand point is regarded as “covered” only if both servers are located within a specified distance.

The preceding models do not consider the system congestion and unavailability of the facilities. Many covering models have also been developed to address the possible congestion condition by providing redundant or back-up coverage. Daskin and Stern (1981) formulated a hierarchical objective LSCP for emergency medical service in order to find the minimum number of vehicles that are required to cover all demand areas while simultaneously maximizing the multiple coverage. Bianchi and Church (1988) proposed an EMS facility model in which they restricted the number of facilities but allowed more than one server at each facility site. Benedict (1983), Eaton et al. (1986), and Hogan and ReVelle (1986) developed MCLP models for emergency service that has a secondary “backup-coverage” objective. The models ensure that a second (backup) facility could be available to service a demand area in case that the first facility is unavailable to provide services. The backup coverage models have been popularly called as BACOP1 (Backup Coverage Problem 1). Since the models of BACOP1 require each demand point to have first coverage which is not necessary for many location problems, Hogan and ReVelle

(1986) further formulated the BACOP2 model which is able to respectively maximize the population that achieve first and second coverage.

Research on emergency service covering models has also been extended to incorporate the stochastic and probabilistic characteristics of emergency situations so as to capture the complexity and uncertainty of these problems. Examples of these stochastic models can be found in recent papers by Goldberg and Paz (1991), ReVelle et al. (1996), and Beraldi and Ruszczynski (2002). There are several approaches to model stochastic emergency service covering problems. The first approach is to use chance constrained models (Chapman and White, 1974). Daskin (1983) used an estimated parameter (q) to represent the probability that at least one server is free to serve the requests from any demand point. He formulated the Maximum Expected Covering Location Problem (MEXCLP) to place P facilities on a network with the goal to maximize the expected value of population coverage. ReVelle and Hogan (1986) later enhanced the MEXCLP and proposed the Probabilistic Location Set Covering Problem (PLSCP). In the PLSCP, an average server busy fraction (q_i) and a service reliability factor (α) are defined for the demand points. Then the locations of facilities are determined such that the probability of service being available within a specified distance is maximized. The MEXCLP and PLSCP later were further modified to tackle other EMS location problems by ReVelle and Hogan (MALP) (1989a), Bianchi and Church (MOFLEET) (1988), Batta et al. (AMEXCLP) (1989), Goldberg et al. (1990), and Repede and Bernardo (TIMEXCLP) (1994). A summary and review to the chance constrained emergency service location models can be found in ReVelle (1989).

Another approach to modeling stochastic EMS covering problems is to use scenario planning to represent possible values for parameters that may vary over the planning horizon in different emergency situations. A compromise decision is made to optimize the expected/worst-case performance or expected/worse-case regret across all scenarios. For example, Schilling (1982) extended the MCLP by incorporating scenarios to maximize the covered demands over all possible scenarios. Individual scenarios are respectively used to identify a range of good location decisions. A compromise decision is made to the final location configuration that is common to all scenarios in the horizon.

One important thrust and cornerstone in location theory is the development and application of the queuing approach in solving EMS location problems. The most well known queuing models for emergency service location problems are the hypercube and approximated hypercube by Larson (1974, 1975), which consider the congestions of the system by calculating the steady-state busy fractions of servers on a network. The hypercube model can be used to evaluate a wide variety of output performance such as vehicle utilization, average travel time, inter-district service performance, etc. Particularly important in the hypercube models is the incorporation of state-dependent interactions among facilities (mobile servers) that preclude applications of traditional location models. Larson (1979) and Brandeau and Larson (1986) later further extended and applied the hypercube models with locate-allocate heuristics for optimizing many realistic EMS systems. For example, these extended models have been successfully used to optimize the ambulance deployment problems in Boston and the EMS systems in New

York. Based on the hypercube queuing model, Jarvis (1977) developed a descriptive model for operation characteristics of an EMS system with a given configuration of resources and a location model for determining the placement of ambulances to minimize average response time or other geographically based variables. Marianov and ReVelle (1996) created a realistic location model for emergency systems based on results from queuing theory. In their model, the travel times or distances along arcs of the network are considered as random variables. The goal is to place limited numbers of emergency vehicles, such as ambulances, in a way as to maximize the calls for service. Queuing models formulating other various theoretical and practical problems have also been reported by Berman and Larson (1985), Batta (1989), and Burwell et al. (1993).

3.2 *P*-Median Models for Emergency Services

Another important way to measure the effectiveness of facility location is by evaluating the average (total) distance between the demand points and the facilities. When the average (total) distance decreases, the accessibility and effectiveness of the facilities increases. This relationship applies to both private and public facilities such as supermarkets, post offices, as well as emergency service centers, for which proximity is desirable. The *P*-median problem, introduced by Hakimi (1964), takes this measure into account and is defined as: determine the location of *P* facilities so as to minimize the average (total) distance between demands and facilities. Later ReVelle and Swain (1970) formulated the *P*-median problem as a linear integer program and used a branch and bound algorithm to solve the problem.

Since its formulation the *P*-median model has been enhanced and applied to a wide range of emergency facility location problems. Carbone (1974) formulated a deterministic *P*-median model with the objective of minimizing the distance traveled by a number of users to fixed public facilities such as medical or day-care centers. Recognizing the number of users at each demand node is uncertain, the author further extended the deterministic *P*-median model to a chance constrained model. The model seeks to maximize a threshold and meanwhile ensure the probability that the total travel distance is below the threshold is smaller than a specified level α . Calvo and Marks (1973) constructed a *P*-median model to locate multi-level health care facilities including central hospitals, community hospitals and local reception centers. The model seeks to minimize distance and user costs, and maximize demand and utilization. Later, the hierarchical *P*-median model was improved by Tien et al. (1983) and Mirchandani (1987) by introducing new features and allowing various allocation schemes to overcome the deficient organization problem across hierarchies. Paluzzi (2004) discussed and tested a *P*-median based heuristic location model for placing emergency service facilities for the city of Carbondale, IL. The goal of this model is to determine the optimal location for placing a new fire station by minimizing the total aggregate distance from the demand sites to the fire station. The results were compared with the results from other approaches and the comparison validated the usefulness and effectiveness of the *P*-median based location model.

One major application of the P -median models is to dispatch EMS units such as ambulances during emergencies. Carson and Batta (1990) proposed a P -median model to find the dynamic ambulance positioning strategy for campus emergency service. The model uses scenarios to represent the demand conditions at different times. The ambulances are relocated in different scenarios in order to minimize the average response time to the service calls. Berlin et al. (1976) investigated two P -median problems to locate hospitals and ambulances. The first problem has a major attention to patient needs and seeks to minimize the average distance from the hospitals to the demand points and the average ambulance response time from ambulance bases to demand points. In the second problem, a new objective is added in order to improve the performance of the system by minimizing the average distance from ambulance bases to hospitals. Mandell (1998) developed a P -median model and used priority dispatching to optimally locate emergency units for a tiered EMS system that consists of advanced life-support (ALS) units and basic life-support (BLS) units. The model can also be used to examine other system parameters including the balance between ALS and BLS units, and different dispatch rules.

Uncertainties have also been considered in many P -median models. Mirchandani (1980) examined a P -median problem to locate fire-fighting emergency units with consideration of stochastic travel characteristics and demand patterns. The author took into account the situations that a facility may not be available to serve a demand and used a Markov process to create a system in which the states were specified according to demand distribution, service and travel time, and server availability. Serra and Marianov (1999) implemented a P -median model and introduced the concept of regret and minmax objectives when locating fire station for emergency services in Barcelona. The authors explicitly addressed in their model the issue of locating facilities when there are uncertainties in demand, travel time or distance. In addition, the model uses scenarios to incorporate the variation of uncertainties and seeks to give a compromise solution by minimizing the maximum regret over the scenarios.

P -median models have also been extended to solve emergency service location problems in a queuing theory context. An example is the stochastic queue median (SQM) model due to Berman et al. (1985). The SQM model seeks to optimally dispatch mobile servers such as emergency response units to demand points and locate the facilities so as to minimize average cost of response.

3.3 P -Center Models for Emergency Services

In contrast to the P -median models which concentrate on optimizing the overall (or average) performance of the system, the P -center model attempts to minimize the worst performance of the system and thus addresses situations in which service inequity is more important than average system performance. In location literature, the P -center model is also referred to as the minimax model since it minimizes the maximum distance between any demand point and its nearest facility. The P -center model considers a demand point is served by its nearest facility and therefore full coverage to all demand points is always achieved. However, unlike the full coverage in the set covering models, which may lead

to excessive number of facilities, the full coverage in the P -center model requires only a limited number (P) of facilities.

The center problem was first posed by Sylvester (1857) more than one hundred years ago. The problem asks for the center of a circle that has the smallest radius to cover all desired destinations. In the last several decades, the P -center model and its extensions have been investigated and applied in the context of locating facilities such as EMS centers, hospitals, fire station, and other public facilities.

In order to locate a given number of emergency facilities along a road network, Garfinkel et al. (1977) examined the fundamental properties of the P -center problem. He modeled the P -center problem using integer programming and the problem was successfully solved by using a binary search technique and a combination of exact tests and heuristics. ReVelle and Hogan (1989b) formulated a P -center problem to locate facilities so as to minimize the maximum distance within which EMS service is available with α reliability. System congestion is considered and a derived server busy probability is used to constrain the service reliability level that must be satisfied for all demands. Stochastic P -center models have also been formulated for EMS location problems. For example, Hochbaum and Pathria (1998) considered the emergency facility location problem that must minimize the maximum distance on the network across all time periods. The cost and distance between locations vary in each discrete time period. The authors used k underlying networks to represent different periods and provided a polynomial time 3-approximation algorithm to obtain the solution for each problem. Talwar (2002) utilized a P -center model to locate and dispatch three emergency rescue helicopters to serve the growing EMS demands from accidents of tourist activities such as skiing, hiking and climbing at the north and south of Alpine mountain ranges. One of the model's aims is to minimize the maximum (worst) response times and the author used effective heuristics to solve the problem.

There are still many other applications and analyses to various P -center models. Readers interested in these applications and their mathematical formulations are referred to Handle (1990), Brandeau et al. (1995), Daskin (2000), and Current et al. (2004).

4 LEMS Facility Location Models

In this section we analyze the unique characteristics that are inherent in the large-scale emergency situations. We then propose a LEMS facility location model that is suited for large-scale emergencies.

4.1 Characteristics of Large-scale Emergency Situations

A major distinction of large-scale emergencies from other regular emergencies is the sudden and tremendous demands for EMS, which overwhelms first responders. Another significant difference is the low frequency of large-scale emergencies, as opposed to other regularly occurring emergencies such as fires and vehicle accidents. The high

demand and low frequency features of the large-scale emergencies require redundant and dispersed placement of EMS facilities so that more medical supplies could be mobilized during emergencies to reduce mortality and morbidity. A dispersed and redundant facility location pattern also helps improve the serviceability and survivability of the facilities because large-scale emergencies such as earthquakes are likely to cause malfunction of some facilities due to road impediment or facility destruction.

Potential demand areas for LEMS need to be categorized in a way different from other regular emergencies. Each demand area has distinct attributes such as population density, economic importance, geographical feature, weather pattern, etc. Therefore, the likelihood for a certain type of large-scale emergency occurring in one area, as well as the corresponding impact level, are different from the other areas. For example, a large seaport may have a relatively high likelihood to receive a dirty bomb terrorist attack since explosives and radioactive materials can be easily hidden in the cargo of a ship; while an upwind district of a populous downtown area may be more likely to receive a biological terrorist attack because winds could spread the biological agents towards a large part of the community. Therefore the facility location pattern should take into account the unique attributes of each demand area and classify the likelihood for a particular emergency to occur in each area.

Another important aspect of LEMS facility locations is the fact that given the occurrence of an emergency at a location, the resources of a number of facilities will be applied to quell the impact of the emergency, not only those that are located closest to the emergency site. This implies that in fact there are different types of coverage, or quality of coverage, which can be classified in terms of the distance (time) between each facility and demand point. Thus a facility that is close to a demand point provides a better quality of coverage to that demand point than a facility located far from that demand point. When locating LEMS it is important to consider adequate location of facilities of various qualities for each demand point. In addition since we are considering a capacitated problem, the number or quantity of facilities of each quality that are located near to each demand point is also important. Furthermore, because the attributes associated to each demand point are distinct, different requirements of facility quantity and quality should be assigned for each demand point. We note there has been some limited work for developing location models that consider the backup (redundant) coverage (Benedict, 1983; Hogan and ReVelle, 1986). However the prior models only allow for at most double coverage of each demand point, which are considered to have identical attributes. As a result the facility quantity and quality of each demand point in these models are limited and uniform. Our modeling framework generalizes the earlier work by allowing for multiple types of facility quantity and quality.

Care should also be taken in dealing with large-scale emergencies that are distinct in nature and thus require different facility deployment strategies. The facility deployment strategies can be generally categorized as two types: (1) proactive facility deployment, and (2) reactive facility deployment. Proactive facility deployment means to pre-position the facilities as well as the medical supplies before an emergency has occurred. This strategy is suited for emergencies that need instantaneous EMS response. For example, a

dirty bomb terrorist attack emergency usually leads to incidents of severe contamination. This type of terrorist attack is overt and the people and environment that are affected need to be treated and decontaminated as soon as possible. Therefore medical supplies should be inventoried a priori at local/regional facilities so that they can be delivered quickly to the demand points. In contrast, a reactive facility deployment does not store large volumes of medical supplies at possible demand points prior to the emergency. Instead, during an emergency the SNS will be called to supply local EMS. The facility location problem thus involves determining local staging centers to receive, repackage and distribute the medical supplies from SNS to the demand points. The reactive facility deployment strategy is particularly applicable to emergencies that have delayed effects and require continuous and large amounts of medical supplies. For example, an anthrax biological warfare agent usually has no immediate warning signs upon release and the lack of warning may lead to continuous and widespread infection of people. A large amount of antibiotics and vaccines may be required to cure and immunize the infected population during such an emergency. However an anthrax terrorist attack has a low occurrence frequency and it is very expensive for any local region to maintain massive medical supplies for such a rare event. A better strategy thus for this emergency is to call SNS and use local staging centers to receive, repackage and distribute the SNS supplies to local areas. There are also emergencies that require a hybrid (both proactive and reactive) facility deployment strategy. For these emergencies, local supplies are needed for immediate use while the SNS is needed to address the massive demand increase for medical supplies.

Moreover, the facility location objective for large-scale emergencies should be carefully defined. A large-scale emergency is bound to impact lives regardless of the solution. Thus care should be taken in prioritizing one solution over another. In this paper we consider different objectives in order to minimize unmet demands and life-loss for different large-scale emergencies.

Finally, the selection of eligible facility sites for large-scale emergencies must consider a different set of criteria that are used for regular emergencies. For instance, the facilities should have easy access to more than one major road/highway including egress and ingress. The sites should be secure and insusceptible from damages caused by the emergencies. In this paper we consider eligible facility sites as given.

4.2 The General LEMS Facility Location Problem

In this section, we propose a general LEMS facility location model that takes into account the characteristics of the large-scale emergencies. The proposed model can be cast as a generalization of the covering, P -median, and P -center models that have been developed for regular EMS facility locations. We consider a set I of demand points and a set J of possible facility locations. Indexed on these sets we define three types of integer variables:

Decision variables:

$$x_j = \begin{cases} 1 & \text{if a facility is placed at } j; \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{ij} = \begin{cases} 1 & \text{if a facility } j \text{ services demand point } i; \\ 0 & \text{otherwise.} \end{cases}$$

$$u_i = \begin{cases} 1 & \text{if demand point } i \text{ is covered;} \\ 0 & \text{otherwise.} \end{cases}$$

The demand for LEMS is uncertain and depends on various factors including the emergency scenario, the impact of the emergency, and the likelihood of the emergency to affect a demand point. To represent the uncertain demand we make use of discrete scenarios from a set K of possible emergency situations. In addition we make use of two parameters: β_{ik} to represent both the likelihood that a certain emergency situation k affects demand point i , and e_{ik} the impact that the emergency situation will have on the population of demand point i , M_i . The likelihood parameter captures the geographical effect of emergency situation k due to location of event, prevailing winds, etc. These likelihoods do not sum to one over all demand points since (1) there can be more than one emergency occurring at the same time, and (2) there can be multiple demand points simultaneously affected by an emergency (e.g. a biological terrorist attack).

Demand parameters:

M_i = the population of demand point i ;

e_{ik} = the impact coefficient for demand point i under large-scale emergency situation k ;

β_{ik} = the likelihood for demand point i to have a large-scale emergency situation k ;

With these parameters, the demand at point i under emergency situation k is represented by $\beta_{ik} \times e_{ik} \times M_i$; this value is also the weight of demand point i in our location models.

Since the different location models (covering, P -median, and P -center) are defined in part by the objective function used, for the moment we represent all models as setting the integer variables x_j , z_{ij} , and u_i so as to optimize a function that represents the efficiency in covering the uncertain demand. We denote this function as $\sigma(x_j, z_{ij}, u_i; \beta_{ik}, e_{ik}, M_i)$.

In addition we consider the following problem parameters:

Input:

Q_i = the minimum number of facilities that must be assigned to demand point i to consider i covered;

p_{jk} = reduction in service capability of facility j under emergency scenario k ;

P = the maximal number of facilities that can be placed in J ;

We can now formulate the general LEMS location model to locate P facilities to address emergency scenario k , requiring that Q_i facilities service demand point i with the same quality. We also consider that the capability of a facility to provide EMS services may be

disrupted during a large-scale emergency due to road impediment and/or the destruction of the facility; hence we use the parameter p_{jk} , a value between 0 and 1, to represent the degree of disruption of facility j under emergency scenario k .

$$\text{Max / Min } \sigma(x_j, z_{ij}, u_i; \beta_{ik}, e_{ik}, M_i) \quad (1)$$

Subject to:

$$\sum_{j \in J} x_j \leq P \quad (2)$$

$$x_j \in \{0,1\} \quad \forall j \in J, \quad (3)$$

$$\sum_{j \in J} z_{ij} p_{jk} \geq Q_i u_i \quad \forall i \in I, \quad (4)$$

$$z_{ij} \leq x_j \quad \forall i \in I, j \in J, \quad (5)$$

$$z_{ij}, u_i \in \{0,1\} \quad \forall i \in I, j \in J, \quad (6)$$

Here constraints (2) and (3) are used to represent that there are P facilities to be located in a set J of possible locations. Constraint (4) indicates that demand point i is considered as covered only if there are more than the required quantity (Q_i) of facilities servicing it. Note that the reduced service capability in each facility has been considered by multiplying z_{ij} by p_{jk} . Finally, constraint (5) shows that a demand point i can only be serviced from facilities j that have been opened; and constraint (6) enforces the integrality of variables z_{ij} and u_i . Note that scenario k is given in the problem above. Thus the solution optimizes the location problem for that particular scenario. If a global solution is desired across multiple scenarios, for instance for a proactive facility deployment, then a regret function defined across all scenarios can be optimized. This concept is illustrated below in Section 4.3. In the rest of this section we show how the covering, P -median, and P -center models are derived from this general LEMS facility location model with appropriate objective function and linear constraints on z_{ij} and u_i .

Consider now the problem with multiple quality-of-coverage requirements at each demand point. Let us assume that at demand point i we must have $Q_i^1, Q_i^2, \dots, Q_i^q$, facilities for each quality from 1 to q , where quality Q_i^1 represents the facilities that are closest to demand point i , Q_i^2 are the facilities located farther than those of quality 1, and so on. If we consider that a facility of quality r is also of quality $r+1, \dots, q$ since it is closer than required for larger quality of coverage, we must have that the requirements satisfy $Q_i^r \leq Q_i^{r+1}$ for all qualities $r = 1, \dots, q-1$. Thus the general LEMS facility location model is modified only on constraint (4) with the following group of constraints:

$$\sum_{j \in J} z_{ij} p_{jk} \geq Q_i^r u_i \quad \forall i \in I, r = 1, \dots, q \quad (4')$$

Below we show how the general LEMS facility location problem leads to covering, P -median, and P -center problems only for the single quality-of-coverage case. The generalization for the multiple quality case is straightforward from the construction above and is left to the reader.

4.3 A Covering Model for LEMS

A covering model is in essence a feasibility problem. Given certain requirements for the quality of coverage we are interested in covering the greatest amount of the demand that can be generated by an emergency. Given that we know emergency scenario k occurred the coverage of the demand is given by $\sum_{i \in I} \beta_{ik} e_{ik} M_i u_i$. Therefore the objective of the

covering model is defined as

$$\text{Maximize } \sigma(x_j, z_{ij}, u_i; \beta_{ik}, e_{ik}, M_i) = \sum_{i \in I} \beta_{ik} e_{ik} M_i u_i.$$

This objective finds an optimal facility location solution (S_k) with value V_k for emergency scenario k . Since different emergency scenarios have different optimal location solutions, a globally optimal location solution (\bar{S}) is needed and it can be obtained by minimizing the regret across all emergency scenarios ($\text{Minimize} \sum_{k \in K} R_k$), where the regret

associated with scenario k is defined by $R_k = \sum_{i \in I} \beta_{ik} e_{ik} M_i u_i - V_k$. Note that in this regret

function, a uniform weight is assigned to all emergency scenarios. Given the fact that the occurrence probabilities and the impact levels of various emergency situations are different, distinct weights (w_k) should be assigned to different emergency scenarios so that more attention could be given to the large-weighted, most likely, emergency situations. Thus the regret function can be re-defined as $\text{Minimize} \sum_{k \in K} w_k R_k$.

Covering models include constraints that limit when a facility can service a demand point. For example, a covering model might allow only facilities located within distance D_i from demand point i to service the demand generated at i . If d_{ij} represents the shortest distance between demand point i and facility location j , then these requirements are represented by the constraints

$$z_{ij} = 0 \text{ if } d_{ij} > D_i. \quad \forall i \in I, j \in J.$$

Let the set $N_i = \{j \mid d_{ij} \leq D_i\}$ be the set of eligible facility sites that can service demand point i . Note that $z_{ij} = 0$ for any $j \notin N_i$, then it is straightforward to show that the LEMS covering problem for a given scenario k is given by

$$\text{Maximize } \sum_{i \in I} \beta_{ik} e_{ik} M_i u_i \quad (7)$$

Subject to:

$$\sum_{j \in N_i} x_j p_{jk} \geq Q_i u_i \quad \forall i \in I, \quad (8)$$

$$\sum_{j \in J} x_j \leq P \quad (9)$$

$$x_j, u_i = \{0, 1\} \quad \forall i \in I, j \in J, \quad (10)$$

4.4 A P -Median Model for LEMS

Both the P -median and P -center problems consider relaxed quality requirements for coverage and seek the facility location solution which achieves the minimum demand-weighted distance between the opened facilities and demand points. This objective is based on the idea that the accessibility and effectiveness of EMS facilities in response to an emergency situation will increase if the distance from the facilities to the demand points decreases. Therefore, since the service distance to demand point i is defined as the sum of distances from demand point i to its servicing facilities, the objective function is simply the sum of the service distances for all demand points, i.e.

$$\text{Minimize } \sum_{i \in I} \sum_{j \in J} \beta_{ik} e_{ik} M_i d_{ij} z_{ij} .$$

Although we have relaxed the quality requirements, (i.e. the variables z_{ij} have no constraints), we do require that each demand point be serviced. In other words, for each demand point i , Q_i facilities are assigned to service it. This is achieved through the constraints

$$u_i = 1 \quad \forall i \in I.$$

The P -median problem for a given scenario k can then be stated as

$$\text{Minimize } \sum_{i \in I} \sum_{j \in J} \beta_{ik} e_{ik} M_i d_{ij} z_{ij} \quad (11)$$

Subject to:

$$\sum_{j \in J} x_j \leq P \quad (12)$$

$$\sum_{j \in J} z_{ij} P_{jk} = Q_i \quad \forall i \in I, \quad (13)$$

$$z_{ij} \leq x_j \quad \forall i \in I, j \in J, \quad (14)$$

$$x_j, z_{ij} = \{0, 1\} \quad \forall i \in I, j \in J, \quad (15)$$

4.5 A P -Center Model for LEMS

In the following, a P -center model is formulated by using the objective of minimizing the maximum service distance for all demand points. The service distance for demand point i is defined as the average distance from demand point i to its nearest Q_i facilities. The P -center model for a given scenario k can be written as the following integer linear program:

$$\text{Minimize } L \quad (16)$$

Subject to:

$$\sum_{j \in J} x_j \leq P \quad (17)$$

$$\sum_{j \in J} z_{ij} p_{jk} = Q_i \quad \forall i \in I, \quad (18)$$

$$z_{ij} \leq x_j \quad \forall i \in I, j \in J, \quad (19)$$

$$L \geq \frac{\sum_{j \in J} \beta_{ik} e_{ik} M_i d_{ij} z_{ij}}{Q_i} \quad \forall i \in I, k \in K, \quad (20)$$

$$x_j, z_{ij} = \{0, 1\} \quad \forall i \in I, j \in J, \quad (21)$$

The only difference with the P -median problem comes from the maximum distance objective, which is represented by objective function (16) and constraint (20).

Note that the P -median and P -center models introduced above consider the case of single quality of coverage. A slight modification in the objective and constraints is needed for the case of multiple quality of coverage. For instance, to formulate the P -center model with two qualities of coverage, the objective (16) and constraints (20) can be re-stated as:

$$\text{Minimize } L_1 + L_2 \quad (16')$$

$$L_r \geq \frac{\sum_{j \in J} \beta_{ik} e_{ik} M_i d_{ij} z_{ij}^r}{Q_i^r} \quad \forall i \in I, k \in K, r = 1, 2 \quad (20')$$

5 Illustrative Examples

In this section we give illustrative examples to show how the proposed models can be used to optimize the facility locations for different large-scale emergencies. The covering model, P -center model and P -median model are respectively used to formulate the location problems for the emergencies including dirty bomb, anthrax and smallpox terrorist attacks. A single quality of coverage is considered in these examples, and furthermore we assume that the reduction of facility service capability caused from these emergencies can be ignored (i.e. $p_{jk} = 1$).

The area in which we consider to locate facilities is Los Angeles (LA) County (Figure 1). We grid the area into square zones using the center of each zone as an aggregated demand point. In this example, we only consider seven demand points (zones), and they are West Hollywood, Downtown, LAX airport, Port of Long Beach, Port of LA, Disneyland, and Rowland Heights.

Furthermore, a number of eligible sites in which the EMS facilities could be placed are identified. We assume that resource limitation allows only 4 local facilities or staging centers to be opened and they can be placed at any of 7 possible eligible facility sites. Table 1 gives the roadway distances between each demand point and each eligible facility site.



Figure 1. Los Angeles County, California

Table 1. Roadway distances between demand points and eligible facility sites

Distances (Miles)	West Hollywood	Downtown	LAX airport	<u>1. Port of Long Beach</u>	Port of LA	Disneyland	Rowland Heights
Site 1	5	4	10	27	25	33	33
Site 2	11	5	5	12	14	16	24
Site 3	4	5	10	30	31	33	35
Site 4	27	13	20	27	33	16	10
Site 5	28	18	10	4	4	20	36
Site 6	20	12	7	4	7	14	32
Site 7	30	20	17	8	12	8	27

Case 1 – A Dirty Bomb Attack

The first emergency we consider is a dirty bomb terrorist attack, which could have a severe local impact (e.g. radiological contamination of the population and environment). A proactive facility deployment strategy is appropriate for this emergency as EMS supplies such as protective equipment and anti-radioactive drugs need to be stored at local facilities. To be better prepared for such an emergency, it is desirable that as much of the population as possible be covered with the required facility quantity and quality. As such in what follows we use the covering model to formulate this facility location problem. Note that in a proactive facility deployment there is no certainty of which emergency scenario will occur, thus the problem should consider a regret objective.

However in this illustrative example we simply assume that we know the emergency scenario (for instance the most likely) and prepare for it.

To provide the input parameters for the model, we first characterize the attributes of each demand point including the emergency occurrence likelihood (β), population (M) and impact coefficient (e). Then the weight ($\beta \times e \times M$) of each demand point is calculated and based on the weight we further derive the parameters of required facility quantity and quality for each demand point (see Table 2). Note that a demand point with a larger weight indicates that it is of relatively high susceptibility to the emergency and thus needs to be covered by more facilities within a short distance. For example, the demand point of the Downtown area has a larger weight, in comparison to the demand point of Disneyland, and therefore it has been assigned with more facilities that are of a better quality.

Table 2. Demand point characterization for dirty bomb emergency

Demand point (I)	Population (M)	Dirty bombing occurrence likelihood (β)	Impact coefficient (e)	Weight ($\beta e M$)	Required facility quantity (Q)	Required facility quality/distance (D)
West Hollywood	76k	Intermediate – high (0.7)	0.7	37.2	2	9 miles
Downtown	94k	High (0.85)	0.8	64.0	3	8 miles
LAX airport	56k	High (0.9)	0.9	45.4	3	9 miles
Port of LA	32k	High (0.9)	0.8	23.0	2	10 miles
Port of Long Beach	28k	High (0.9)	0.8	20.2	2	12 miles
Disneyland	34k	Intermediate (0.5)	0.5	8.5	1	15 miles
Rowland Heights	8k	Low (0.3)	0.3	0.72	1	15 miles

Based on these input parameters, the covering model proposed in Section 4.3 could be applied to determine the facility locations for medical supply storage. We optimally solve the problem using CPLEX (a commercially available optimization software package) and the result suggests the facilities should be placed at sites 1, 2, 3, and 6. This result is reasonable since the demand points with large weights including West Hollywood, Downtown, LAX airport, and Port of Long Beach are covered by sufficient facilities that are of the required quality. The population that has been protected with the required facility quantity and facility quality accounts for 78 percent of the population considered.

Case 2 – An Anthrax Terrorist Attack

The second emergency we consider is an anthrax terrorist attack. Anthrax is an acute infectious disease caused by the spore-forming bacterium. There are several types of anthrax infections (cutaneous, inhalation, and gastrointestinal) and each requires different vaccines and antibiotics to cure infected people and immunize the high-risk population. During an anthrax emergency, the federal government needs to carefully investigate the

specific anthrax infection type and then deploy the appropriate SNS supplies for local EMS services. Therefore in this example we use a reactive facility deployment strategy to optimize the locations of facilities (local staging centers) for receiving, repackaging, and distributing the medical supplies from the SNS. We assume that the eligible facility sites in Figure 1 represent the sites that can be used as the staging areas. Another important feature of anthrax is that it does not transmit through person-to-person contact, and hence terrorists may send infectious materials to multiple places to increase the threats (one example is the anthrax emergency after September 11). Since it is difficult to predict where the infectious material will be sent to, we use the P -center model to determine the facility locations with the objective that the worst case can be avoided, i.e. the maximum demand-weighted distance between the staging centers and the demand points can be minimized.

Similar to the preceding example, we first categorize the attributes of the demand points and calculate the weight for each point. Based on the weight a required facility quantity is specified for each demand point, as shown in Table 3. In both the P -center and P -median problems a demand point is serviced by the nearest required quantity of facilities, hence we do not explicitly assign a facility quality requirement at each demand point.

Table 3. Demand point characterization for anthrax emergency

Demand point (I)	Population (M)	Anthrax Bioterrorism occurrence likelihood (β)	Impact coefficient (e)	Weight ($\beta e M$)	Required facility quantity (Q)
West Hollywood	76k	High (0.8)	0.6	36.4	2
Downtown	94k	High (0.85)	0.6	48.0	3
LAX airport	56k	High (0.8)	0.7	31.4	2
Port of LA	32k	Low (0.4)	0.3	3.8	1
Port of Long beach	28k	Low (0.4)	0.3	3.4	1
Disneyland	34k	Intermediate (0.6)	0.5	10.2	1
Rowland Heights	8k	Low (0.3)	0.3	0.72	1

With these input parameters, the P -center model proposed in Section 4.4 is used to determine the facility locations for the anthrax emergency. The result obtained from the model suggests the selection of sites 1, 2, 3, and 7 as the facility locations. Similar to the preceding example, the solution also gives emphasis to the large-weighted demand points (Downtown, West Hollywood, Airport, etc.) by placing the facilities at sites 1, 2, and 3, which are near to these demand points. However, different from the solution in the preceding example, the P -center model selects site 7, instead of site 6, as one facility location. This is because site 7 is relatively close to the light-weighted demand points (Disneyland and Rowland Heights). The placement of a facility at site 7 could improve the facilities' worst performance and the light-weighted demand points can also be better serviced.

Case 3 – A Smallpox Terrorist Attack

The third emergency we consider is a smallpox terrorist attack. The smallpox disease is different from anthrax in that it can be transmitted through person-to-person contact, and thus it can spread much faster than the anthrax disease. First responders (e.g. fire, police, medical personnel, etc.) need instantaneous vaccination once a smallpox emergency is detected to remain effective during the emergency. Therefore medical supplies need to be stored at local facilities for immediate use by the first responders. Furthermore, during a smallpox emergency mass vaccination is often necessary, which requires tremendous medical supplies, hence the SNS would be requested to meet this massive demand. As such a hybrid facility deployment strategy is suitable for a smallpox emergency. The locations of facilities need to be determined not only for storing the medical supplies at a local level but also for receiving and distributing the SNS from the federal government.

The locations of local medical supply storages for the first responders can be determined by using the P -center model, which has been illustrated in the preceding example. We will next concentrate on the determination of the local staging areas for the distribution of the SNS. Because the medical supplies need to be distributed to all demand points, the P -median model proposed in Section 4.5 is appropriate since its objective is to minimize the total distance between all the demand points and their service staging centers. To provide the input parameter to the model, the attributes and the required facility quantity for each demand point are characterized in Table 4. Note that the emergency occurrence likelihood and the impact coefficient for all demand points have been set to 1 because all the population at every demand point must be serviced.

Table 4. Demand point characterization for smallpox emergency

Demand point (J)	Population (M)	Smallpox occurrence likelihood (β)	Impact coefficient (e)	Weight ($\beta e M$)	Required facility quantity (Q)
West Hollywood	76k	1	1	76	3
Downtown	94k	1	1	94	4
LAX airport	56k	1	1	56	3
Port of LA	32k	1	1	32	2
Port of Long beach	28k	1	1	28	2
Disneyland	34k	1	1	34	2
Rowland Heights	8k	1	1	8	1

Based on these parameters the P -median model is used to determine the locations of the local staging centers. The result obtained from the model suggests the selection of sites 1, 2, 3, and 6. Similar to the last two examples, the sites 1, 2 and 3 are selected because they are close to the large-weighted demand points and thus contribute to the objective optimization. Note that site 6 is reselected this time (site 7 is selected in the anthrax example) and this is because site 6 is closer to the demand points of LAX airport and the Ports of LA and Long Beach. Therefore the total distance can be shortened by selecting site 6 instead of site 7. Also note in this solution the demand points of Disneyland and Rowland Heights are relatively weakly serviced, but the population of these two demand points accounts for only 12 percent of the considered population.

6 Conclusions and Future Work

In the past, there has been little attention to the LEMS location problems in the research community. This paper has two primary goals. The first is to review different location models that are commonly used to optimize the facility locations for regular emergencies such as hospitals, fire stations, ambulances, EMS centers, etc. Different models including covering, P -median, and P -center are reviewed. The second goal is to analyze the characteristics of large-scale emergencies and propose tailored location models that take into account the unique characteristics of these emergencies. LEMS location models with different objectives have been formulated. Illustrative examples have been given to show how the proposed models can be used to determine the LEMS facilities locations for different large-scale emergencies.

The illustrative examples developed in this paper were of relatively small size so optimal solutions could be readily found using commercially available optimization software. However, for modeling more realistic and larger scenarios, the problem size of the model will increase significantly so that it becomes computationally prohibitive to obtain an optimal solution. For example, dividing the Los Angeles County area into 5 by 5 square miles zones would result in over 200 demand points. In this case, heuristics are required to obtain a near optimal solution. Our future research direction will focus on developing efficient heuristics which can identify near optimal solutions.

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References

- [1] Batta, R. (1989). A queueing-location model with expected service time-dependent queueing disciplines. *European Journal of Operational Research*, pp. 192-205.
- [2] Batta, R. and Berman, O. (1989). A location model for a facility operating as an $M/G/k$ Queue. *Networks*, 19, pp. 717-729.
- [3] Batta, R., Dolan, J. and Krishnamurthy, N. (1989). The Maximal Expected Covering Location Problem: Revisited. *Transportation Science*, 23, pp.277-287.
- [4] Benedict, J. (1983). Three hierarchical objective models which incorporate the concept of excess coverage for locating EMS vehicles or hospitals. MSc thesis, Northwestern University.

- [5] Beraldi, P. and Ruszczynski, A. (2002). A branch and bound method for stochastic integer problems under probabilistic constraints. *Optimization Methods and Software*, 17, pp. 359-382.
- [6] Berlin, G., ReVelle, C. and Elzinga, J. (1976). Determining ambulance-hospital locations for on-scene and hospital services. *Environment and Planning A*, 8, pp. 553-561.
- [8] Berman, O. and Larson, R.C. (1985). Optimal 2-facility network districting in the presence of queuing. *Transportation Science*, 19, pp. 261-277.
- [9] Berman, O., Larson, R.C. and Chiu, S.S. (1985). Optimal server location on a network operating as an M/G/1 queue. *Operations Research*, 33, pp. 746-771.
- [11] Bianchi, C. and Church, R. (1988). A hybrid FLEET model for emergency medical service system design. *Social Sciences in Medicine*, pp. 163-171.
- [12] Brandeau, M.L., Chiu, S.S., Kumar, S. and Grossman, T.A. (1995). Objective in location problems. In: Drezer, Z. (ed.), *Facility Location: A Survey of Applications and Methods*, Springer-Verlag, New York.
- [13] Brandeau, M. and Larson, R. C. (1986). Extending and applying the hypercube queueing model to deploy ambulances in Boston. In: Swersey, A. and Ignall, E. (ed.), *Delivery of Urban Services*. New York: North Holland.
- [14] Burwell, T., Jarvis, J. and McKnew, M. (1993). Modeling co-located servers and dispatch ties in the hypercube model. *Computers and Operations Research*, pp. 113-119.
- [15] Calvo, A. and Marks, H. (1973). Location of health care facilities: An analytical approach. *Socio-Economic Planning Sciences*, 7, pp. 407-422.
- [16] Campbell, J.F. (1994). A survey of network hub location. *Studies in Locational Analysis*, 6, pp.31-49.
- [17] Carbone, R. (1974). Public facility location under stochastic demand, *INFOR*, 12.
- [18] Carson, Y. and Batta, R. (1990). Locating an ambulance on the Amherst campus of the State University of New York at Buffalo. *Interfaces*, 20, pp.43-49.
- [19] Chapman, S.C. and White, J.A. (1974). Probabilistic formulations of emergency service facilities location problems. *Paper Presented at the ORSA/TIMS Conference*, San Juan, Puerto Rico.
- [20] Chen, R. and Handler, G.Y. (1993). The conditional P -center problem in the plane. *Naval Research Logistics*, 40, pp. 117-127.

- [21] Chung, C.H. (1986). Recent applications of the maximal covering location planning (M.C.L.P.) model. *Journal of the Operational Research Society*, 37, pp.735-746.
- [22] Church, R. and ReVelle, C. (1974). The maximal covering location problem. *Papers of the Regional Science Association*, 32, pp. 101-118.
- [23] Current, J., Daskin, M. and Schilling, D. (1995). Discrete network location models. In: Drezner, Z. and Hamacher, H.W. (ed.), *Facility Location: Applications and Theory*, Springer-Verlag, New York.
- [24] Daskin, M. (1983). The maximal expected covering location model: Formulation, properties and heuristic solution. *Transportation Science*, 17(1), pp. 48-70.
- [25] Daskin, M.S. (1995). *Network and discrete location: Models, algorithms and applications*. John Wiley, New York.
- [26] Daskin, M.S. (2000). A new approach to solving the vertex P -center problem to optimality: Algorithm and computational results. *Communications of the Japanese OR Society*, 9, pp. 428-436.
- [27] Daskin, M.S. and Stern, E.H. (1981). A hierarchical objective set covering model for emergency medical service vehicle deployment. *Transportation Science*, 15, pp. 137-152.
- [28] Eaton, D.J., Daskin, M.S., Simmons, D., Bulloch, B. and Jansma, G. (1985). Determining emergency medical deployment in Austin, Texas. *Interfaces*, 15(1), pp. 96-108.
- [29] Eaton, D., Hector, M., Sanchez, V., Lantigua, R. and Morgan, J. (1986). Determining ambulance deployment in Santa Domingo, Dominican Republic. *Journal of the Operational Research Society*, 37, pp.113-126.
- [30] Erlenkotter, D. (1981). A comparative study of approaches to dynamic location problems. *European Journal of Operational Research*, 6, pp. 133-143.
- [31] Gao, L.L. and Robinson Jr., E.P. (1994). A dual-based optimization procedure for the two-echelon incapacitated facility location problem. *Naval Research Logistics*, 39, pp. 191-212.
- [32] Garfinkel, R. S., Neebe, A. W. and Rao, M. R. (1977). The m -center problem: Minimax facility location. *Management Science*, 23, pp. 1133-1142.
- [33] Goldberg, J., Dietrich, R., Chen, J.M. and Mitwasi, M.G. (1990). Validating and applying a model for locating emergency medical services in Tucson, AZ. *European Journal of Operational Research*, 49, pp. 308-324.

- [34] Goldberg, J. and Paz, L. (1991). Locating emergency vehicle bases when service time depends on call location. *Transportation Science*, pp. 264-280.
- [35] Hakimi, S.L. (1964). Optimum locations of switching centers and the absolute centers and medians of a graph. *Operations Research*, 12, pp.450-459.
- [36] Hamacher, H.W. and Nickel, S. (1998). Classification of location models. *Location Science*, 6, pp. 229-242.
- [37] Handler, G. Y. (1990). *P*-center problems. In: Mirchandani, P.B. and Francis R. L. (ed.), *Discrete Location Theory*, John Wiley, New York.
- [38] Hochbaum, D.S. and Pathria, A. (1998). Locating centers in a dynamically changing network and related problems. *Location Science*, 6, pp. 243-256.
- [39] Hogan, K. and ReVelle, C. (1986). Concepts and applications of backup coverage. *Management Science*, 32, pp.1434-1444.
- [40] Jarvis, J. P. (1977). Models for the location and dispatch of emergency medical vehicles. In: Willemain, T. R. and Larson, R. C. (ed.), *Emergency Medical Systems Analysis*. Lexington, MA: Lexington Books.
- [41] Klose, A. and Drexl, A. (2004). Facility location models for distribution system design. *European Journal of Operational Research*, 162, pp. 4-29.
- [42] Larson, R.C. (1974). A hypercube queuing model for facility location and redistricting in urban emergency services. *Computers and Operations Research*, 1, pp. 67-95.
- [43] Larson, R.C. (1975). Approximating the performance of urban emergency service systems. *Operations Research*, pp. 845-868.
- [44] Larson, R. C. (1979). Structural system models for locational decisions: an example using the hypercube queueing model. In: Haley, K. B. (ed.), *Operational Research '78, Proceedings of the Eighth IFORS International Conference on Operations Research*, North-Holland Publishing Co., Amsterdam, Holland.
- [45] Mandell, M.B. (1998). A *P*-median approach to locating basic life support and advanced life support units. *Presented at the CORS/INFORMS National Meeting*, Montreal, April, 1998.
- [46] Marianov, V. and ReVelle, C. (1992). The capacitated standard response fire protection siting problem: Deterministic and probabilistic models. *Annals of Operations Research*, pp. 303-322.

- [47] Marianov, V. and ReVelle, C. (1996). The queueing maximal availability location problem: A model for the siting of emergency vehicles. *European Journal of Operational Research*, 93, pp.110-120.
- [48] Mirchandani, P.B. (1980). Locational decisions on stochastic networks. *Geographical Analysis*, 12, pp. 172-183.
- [49] Mirchandani, P.B. (1987). Generalized hierarchical facility location. *Transportation Science*, 21, pp. 123-125.
- [50] Owen, S.H. and Daskin, M.S. (1998). Strategic facility location: A review. *European Journal of Operational Research*, 111, pp. 423-447.
- [51] Paluzzi, M. (2004). Testing a heuristic P -median location allocation model for siting emergency service facilities. *Paper Presented at the Annual Meeting of Association of American Geographers*, Philadelphia, PA.
- [52] Pirkul, H. and Schilling, D. (1991). The capacitated maximal covering location problem with capacities on total workload. *Management Science*, 37, pp. 233-248.
- [53] Plastria, F. (1987). Solving general continuous single facility location problems by cutting planes. *European Journal of Operation Research*, 29, pp. 98-110.
- [54] Plastria, F. (1997). Profit maximising single competitive facility location in the plane, *Studies in Locational Analysis*, 11, pp. 115-126.
- [55] Repede, J.F. and Bernardo, J.J. (1994). Developing and validating a decision support system for locating emergency medical vehicles in Louisville, Kentucky. *European Journal of Operational Research*, 40, pp. 58-69.
- [56] ReVelle, C. (1989). Review, extension and prediction in emergency service siting models. *European Journal of Operational Research*, 40, pp. 58-69.
- [57] ReVelle, C. and Hogan, K. (1986). A reliability constrained siting model with local estimates of busy fractions. *Environment and Planning*, B15, pp. 143-152.
- [58] ReVelle, C. and Hogan, K. (1989a). The maximum availability location problem. *Transportation Science*, 23, pp. 192-200.
- [59] ReVelle, C. and Hogan, K. (1989b). The maximum reliability location problem and a-reliable p -center problem: Derivatives of the probabilistic location set covering problem. *Annals of Operations Research*, 18, pp. 155-174.
- [60] ReVelle, C., Schweitzer, J. and Snyder, S. (1996). The maximal conditional covering problem. *INFOR*, 34, pp. 77-91.

- [61] ReVelle, C. and Swain, R.W. (1970). Central facilities location. *Geographical Analysis*, 2, pp. 30-42.
- [62] Rosing, K., Hillsman, E., and Rosing-Vogelaar, H. (1979). The robustness of two common heuristics for the P -median problem. *Environment and Planning A*, 11, pp. 373-380.
- [63] Schilling, D.A. (1982). Strategic facility planning: The analysis of options. *Decision Sciences*, 13, pp. 1-14.
- [64] Schilling, D., Elzinga, D., Cohon, J., Church, R. and ReVelle, C. (1979). The TEAM/FLEET models for simultaneous facility and equipment siting. *Transportation Science*, 13, pp. 163-175.
- [65] Serra., D and Marianov, V. (1999). The P -median problem in a changing network: The case of Barcelona. *Location Science*, 4, 1999.
- [66] Sylvester, J.J. (1857). A question in the geometry of situation. *Quarterly Journal of Pure and Applied Mathematics*, 1, pp. 79.
- [67] Talmar, M. (2002). Location of rescue helicopters in South Tyrol. *Paper Presented at 37th Annual ORSNZ Conference*, Auckland, New Zealand.
- [68] Tien, J.M., El-Tell, K. and Simons, G. (1983). Improved formulations to the hierarchical health facility location-allocation problem. *IEEE Transactions in Systems Management and Cybernetics*, SMC-13, pp. 1128-1132.
- [69] Toregas, C., Swain, R., ReVelle, C. and Bergman, L. (1971). The location of emergency service facility. *Operations Research*, 19, pp. 1363-1373.
- [70] White, J. and Case, K. (1974). On covering problems and the central facility location problem. *Geographical Analysis*, 281, 1974.